Making Napkins

Mathematical Goals

This lesson unit is intended to help you assess how well students are able to:

- Interpret a situation and represent the variables within the situation mathematically.
- Select appropriate mathematical methods to use.
- Explore possible solutions and determine whether they satisfy constraints.
- Communicate their reasoning clearly.

Common Core State Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:

3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

This lesson also asks students to select and apply mathematical content from across the grades, including the
content standards:

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100.

Introduction

This lesson is designed to enable students to develop strategies for solving real-life problems. .

- Before the lesson students attempt to solve the problem individually. You then review their work and formulate questions for students to answer in order to improve their solutions.
- At the start of the lesson, students work alone answering your questions.
- Students are then grouped and engage in a collaborative discussion of the same task.
- In the same small groups, students are given some sample solutions to evaluate and comment on.
- In a whole class discussion, students explain and compare the alternative solution strategies they have seen and used.
- Finally, students review what they have learned.

Materials required

- For each individual student, you will need a copy of the assessment task: *Making Napkins,* and the questionnaire *How Did You Work?*
- Each small group of students will need a large sheet of paper, and copies of all the *Sample responses to discuss.*
- Graph paper should be kept in reserve and used only when requested.
- There are some projector resources to support class discussions.

Time needed

Approximately 15 minutes before the lesson, a one-hour lesson and a follow-up homework. Timings given are only approximate. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task*: Making Napkins* (15 minutes)

Set this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task *Making Napkins.*

> *Read through the questions and try to answer them as carefully as you can. Show all your work, so that I can understand your reasoning. As well as trying to solve the problem, I want you to see if you can present your work in an organized and clear way.*

It is important that students are allowed to answer the questions without your assistance, as far as possible.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Students who sit together often produce similar answers, and then when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a selection of these questions on each piece of student work. If you do not have time, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.

Suggested lesson outline

Introduction: Making Napkins (10 minutes)
Return the assessment task to the students.

If you have not added questions to individual pieces of work, then write your list of questions on the board. Students are to select questions appropriate to their own work, and spend a few minutes answering them.

> *Recall what we were looking at in a previous lesson. What was the task?*
> *I have read your solutions, and I have some questions I would like you to think about. At this point you do not need to do the math, but write a few sentences describing how you could improve your work.*

This is an opportunity for students to review their work.

While they are doing this listen and watch students carefully. Are students able to reflect on their own work and suggest useful, detailed improvements? Ask questions that help students to clarify their thinking.

Collaborative work (20 minutes)
Organize the class into small groups of two or three students.
Give each group a fresh piece of paper.

Deciding on a Strategy
Ask students to share their ideas about the task, and plan a joint solution.

> *I want you to share your work with your group.*
> *Take turns to explain how you did the task and how you now think it could be improved.*
> *Listen carefully to any explanation. Ask questions if you don't understand or agree with the method.*
> *[You may want to use some of the questions I have written on the board.]*
> *I want you to plan a joint approach that is better than your separate solutions.*

Once students have evaluated the relative merits of each approach ask students to write their strategy on one side of their sheet of paper.

Slide 1 of the projector resource summarizes these instructions.

Implementing the Strategy
Students are now to write their joint solution on the second side of their sheet of paper.

While students work in small groups you have two tasks, to note different student approaches to the task and to support student problem solving.

You can then use this information to focus a whole class discussion towards the end of the lesson. Note any common mistakes. For example; are students consistently using all the given information? Which math do they choose to use? How do they use it? Attend in particular to the students' mathematical decisions. Do they track their progress in use of their chosen mathematics? Do they notice if they have chosen a strategy that does not seem to be productive? If so, what do they do?
Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions that help students to clarify their thinking, promote further progress and encourage students to develop self regulation and error detection skills.

Collaborative analysis of Sample responses to discuss (20 minutes)
After students have had sufficient time to solve the problem, give each group of students copies of all *Sample responses to discuss* and ask for written comments. This task gives students an opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy.

> *Imagine you are the teacher and have to assess students response to the second question. Correct the work and write comments on the accuracy and organization of each response.*

To encourage students to do more than check to see if the answer is correct, ask them to answer the questions below each sample piece of work. Encourage students to focus on the math of the student work, not whether the student has neat writing etc. There may not be time for all groups to look at all solutions, and it is not essential that they do. If so, be selective about what you hand out. For example, groups that have successfully completed the task using one method will benefit from looking at different approaches. Other groups that have struggled with a particular approach may benefit from seeing a student version of the same strategy.

During the small group work, support the students as before. Also, check to see which of the explanations students find more difficult to understand. Note similarities and differences between the sample approaches and those the students took in the small group work.

Whole class discussion (10 minutes)
Now hold a whole class discussion to consider the different approaches used in the sample work. Focus the discussion on parts of the small group tasks students found difficult. Ask the students to compare the different solution methods.

> *Which approach did you like best? Why?*
> *Which approach did you find most difficult to understand?*

To critique the different strategies use the questions on the sheets *Sample responses to discuss* and Slides 2 - 5 of the projector resource.

Review solutions to *Making Napkins* (10 minutes)
Give the questionnaire *How Did You Work?* to each student.
Ask students to and complete the review questionnaire.
Some teachers set this task as homework. The questionnaire may help student monitor and review their progress during and at the end of an activity.

Making Napkins

A piece of cloth is 56 centimeters long and 84 centimeters wide.

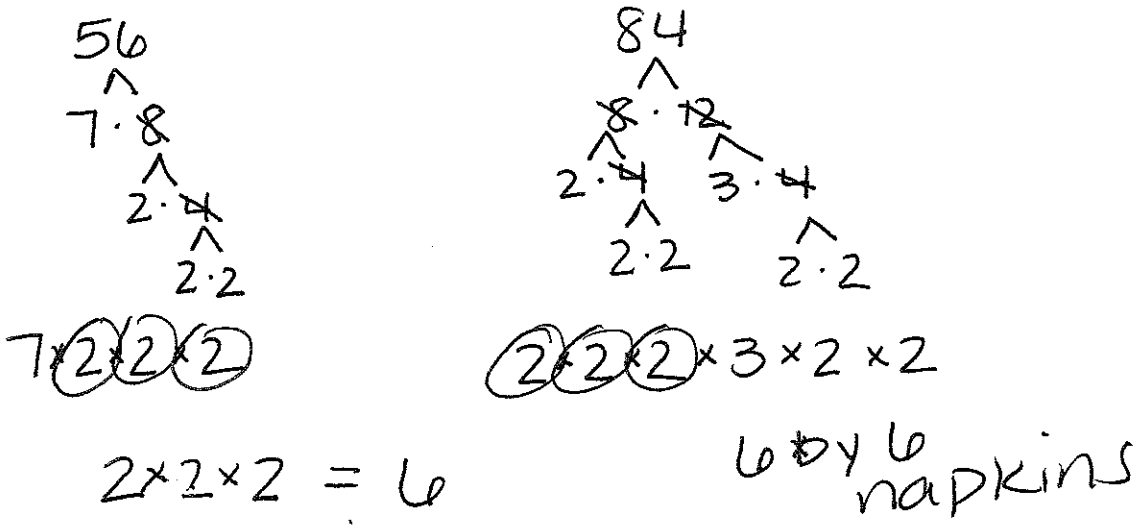
We would like to cut *square* napkins of the same size from this cloth.

We would like to use all of the cloth.

We would like to make the square napkins as large as possible.

What would be the length in centimeters of the side of each napkin?

Sample responses to discuss – Danny



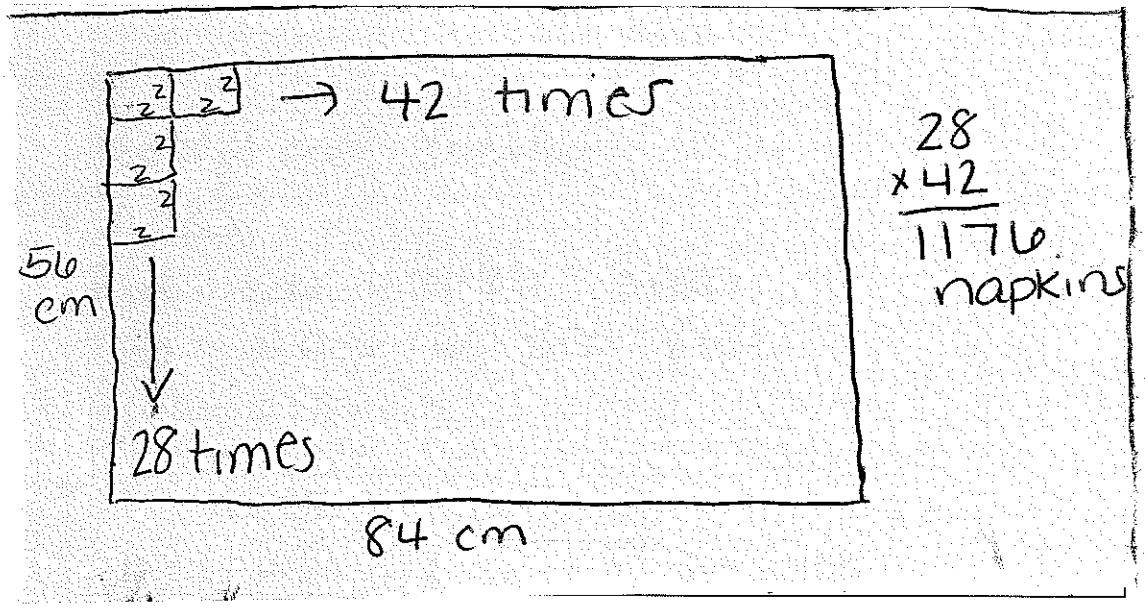
What Math did Danny do well?

Can you find two arithmetical errors in Danny's work?

Do these errors effect his conclusion?

In what ways might the work be improved?

Sample responses to discuss - Jeremiah

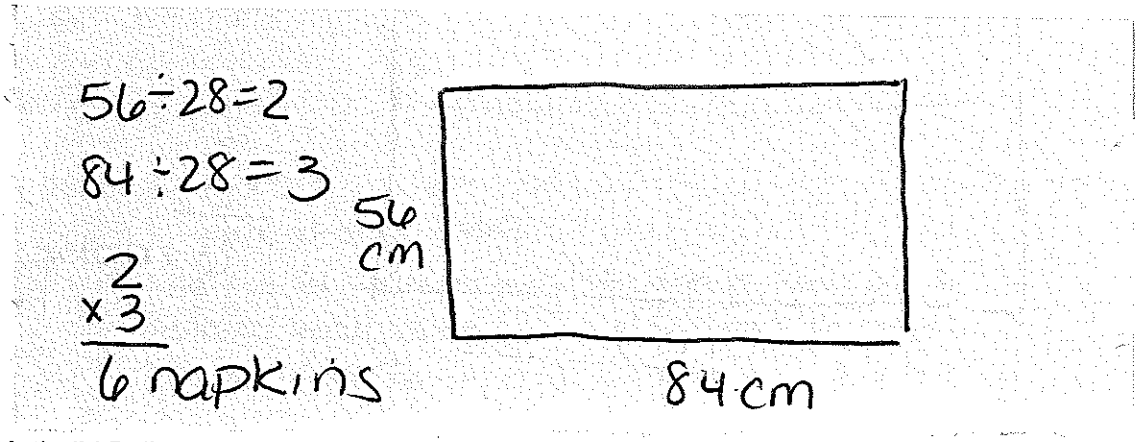


What Math did Jeremiah do well?

Is Jeremiah's solution complete?

In what ways might the work be improved?

Sample responses to discuss - Bella



What Math did Bella do well?

Is Bella's reasoning correct? Explain your answer

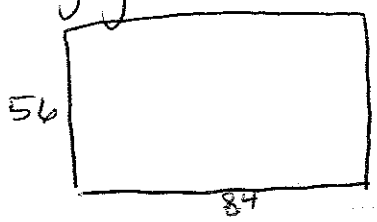
In what ways might the work be improved?

Sample responses to discuss – Tanya

56 - (1), 56, (2), (28), (4), (14), (7), 8

84 - (1), 84, (2), 42, 3, (28), (4), 21, 6, (14), (7), 12

28 is the biggest so 28 by 28



What Math did Tanya do well?

What is not clear about the Tanya's answer?

In what ways might the work be improved?

How Did You Work?

Tick the boxes and complete the sentences that apply to your work.

1. Our group work was better than my own work ☐

   Our joint solution was better because .....

   .....

2. We checked our solution ☐

   We checked our solution by .....

   .....

3. Our solution is similar to one of the sample responses ☐ **OR** Our solution is different from **all** of the sample responses ☐

   Our solution is similar to ..... *(add name of the student)*

   I prefer **our solution / the student's solution** *(circle)*

   This is because .....

   .....

   .....

   Our solution is different from all of the sample responses

   because .....

   .....

   .....

   .....

4. If you taught this lesson to another class, what advice would you give them about potential pitfalls?

   .....

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