Integer Spinner Game

Mathematical goals

This lesson is intended to help you assess how well students are able to use integer operations as well as theoretical probability models and experimental frequencies. In particular this lesson aims to identify and help students with difficulties:

- Adding and multiplying integers
- Choosing an appropriate, systematic way to determine & organize possible outcomes in a probability model
- Comparing theoretical and experimental probabilities

Common Core State Standards

This lesson involves a range of *mathematical practices,* with emphasis on:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
7. Look for and make use of structure.

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

7.NS **Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**

7.SP **Investigate chance processes and develop, use, and evaluate probability models.**

This lesson is structured in the following way:

• Before the lesson, students attempt the *Your Task* section individually. You then review their work and formulate feedback questions.
• At the start of the lesson, students work collaboratively, in small groups to complete the first *Group Task*, which is to critique examples of other students' work on the *Your Task* section.
• In the same small groups, students work collaboratively to complete the second *Group Task*, which is to play enough rounds of t the Integer Spinner Game to determine the experimental probability. Throughout their work, they justify and explain their decisions to peers.
• In a whole-class discussion, students explain and compare the alternative approaches they have seen and used to determine the theoretical & experimental probabilities.
• Finally, students work alone again to complete the Post-Task Reflection about their group experiences with the Integer Spinner Game.

Materials required

- Each individual student will need a copy of the handout *Integer Spinner Game – Game Rules & Your Task* and the *Post-Task Reflection: Integer Spinner Game*.
- Each small group of students will need a copy of *Integer Spinner Game Group Tasks, student responses for Alisha, Jim, & Chip*, and two paperclips to use for the spinners.

Time needed

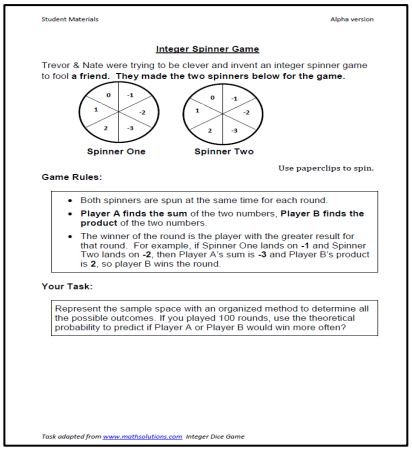
Approximately fifteen minutes before the lesson, a one-hour lesson, and ten minutes in a follow-up lesson. All timings are approximate. Exact timings will depend on the needs of the class.

Before the lesson

Have the students do the Your Task section a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Integer Spinner Game – Game Rules & Your Task*. Introduce the task briefly and help the class to understand the problem and its context.

Spend fifteen minutes on your own, answering the Your Task section of the Integer Spinner Game.
Show your work. Don't worry if you can't figure it out. There will be a lesson on this material [tomorrow] that will help you improve your work. Your goal is to be able to answer this question with confidence by the end of that lesson.



Student Materials — Alpha version

Integer Spinner Game

Trevor & Nate were trying to be clever and invent an integer spinner game to fool **a friend. They made the two spinners below for the game.**

Use paperclips to spin.

Game Rules:

- Both spinners are spun at the same time for each round.
- **Player A finds the sum** of the two numbers, **Player B finds the product** of the two numbers.
- The winner of the round is the player with the greater result for that round. For example, if Spinner One lands on **-1** and Spinner Two lands on **-2**, then Player A's sum is **-3** and Player B's product is **2**, so player B wins the round.

Your Task:

Represent the sample space with an organized method to determine all the possible outcomes. If you played 100 rounds, use the theoretical probability to predict if Player A or Player B would win more often?

Task adapted from www.mathsolutions.com Integer Dice Game

It is important that students answer the question without assistance, as far as possible. Students who sit together often produce similar answers, and then, when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually.

Assessing students' responses

Collect students' responses to the Your Task section. Make some notes on what their work reveals about their current levels of understanding, and their different problem solving approaches. We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the chart on the next page.

We suggest that you write a list of your own questions, based on your students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help the majority of students. These can be written on the board at the end of the lesson.

Common Issues	Suggested questions and prompts
Student makes calculation errors calculating sums or products spinner integers.	• *What math did you use to arrive at your solution?* • *What patterns do you notice when you add negatives versus multiplying negatives?*
Student work is unsystematic.	• *How do you know you have considered all the possible outcomes?* • *How can you organize your work?*
Student confuses expected & experimental probability	• *If there are more outcomes which favor one player over another does that guarantee that player will always win? How do you know?*
Student writes answer without explanation.	• *How could you explain/show how you reached your conclusions so that someone in another class understands?* • *How can you use numbers, words, or diagrams to describe how you know you have all the possible outcomes?*
Student correctly identifies all the possible outcomes in the integer spinner game.	• *Think of another model you could use to represent the expected probability. Is this model better or worse than your original one? Explain your answer.*

Suggested lesson outline

First Collaborative activity (20 minutes)

Return your students' work on the *Integer Spinner Game* – *Game Rules & Your Task*. Ask students to re-read both the *Your Task* problem and their solutions.

Organize the students into small groups of two or three. In trials, teachers found keeping groups small helped more students play an active role.

Give each small group of students a copy of the *Integer Spinner Game Group Tasks,* and the *student responses* for *Alisha, Jim, & Chip*. Display the following Group Task A and questions:

Group Task A: Analyze the probability models in the student work samples and make suggestions about how each can be improved. As a group come to an agreement about the expected probability for Player A and Player B.

For each piece of student work answer the following as a group:
What Math did the student do well?
Is the student's model complete?
In what ways might the work be improved?

This analysis task will give students an opportunity to evaluate a variety of alternative approaches to the task, without providing a complete solution strategy.

During small-group work, support student thinking. Check to see which of the explanations students find more difficult to understand. Identify one or two of these approaches to discuss in the plenary discussion. Note similarities and differences between the sample approaches.

Second Collaborative activity (20 minutes)

As collaborative groups complete Task A, direct them to Group Task B:

*Group Task B: With your group, conduct an experiment by playing the **Integer Spinner Game** for as many rounds as you decide are necessary**.** Compare the frequencies you generate to your probability model and determine if your experimental probability matches what you expected to happen. Is this a fair game?*

During small-group work, support student thinking as before. Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

Plenary whole-class discussion –

Compiling experimental data (10 minutes)

Organize a whole-class discussion about the experimental data the collaborative groups have gathered. Have each small group of students share their game results including the number of rounds they played and why they chose that number of rounds. As groups share their results, compile the total class data for all rounds played.

How many rounds did your groups play of the Integer Spinner Game?
Why did you chose to play that number of rounds?
How do the compiled class experimental results compare to your individual group data?

Making connections between experimental & theoretical probability (10 minutes)

Next have a whole class discussion to consider different approaches for determining the expected probability models and how those compare to the experimental data. The intention is for you to focus on getting students to understand the connections between the different probability models and how more experimental trials result in data closer to what is expected.

Let's stop and talk about different approaches for the expected probabilities.
Which student work samples did you like best? Why?
Which did you find most difficult to understand?
How close were your experimental probabilities to your expected models?
How close where our complied class data to your expected models?
Read through your original responses and think about what you have learned this lesson.

Review solutions to *Spinner Game Your Task* (10 minutes)

If you are running out of time, you could schedule this activity for the next lesson.

Make sure students have their original individual work on the *Integer Spinner Game – Game Rules & Your Task* on hand. Give them a copy of the ***Post-Task Reflection*: Integer Spinner Game** **How You Work** sheet and ask them to complete all sections of the questionnaire reflection and explain their reasoning for their answers.

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This Formative Assessment Lesson was created around a task taken from
MathematicsSolutions.com – Marilyn Burns - Integer Dice Game

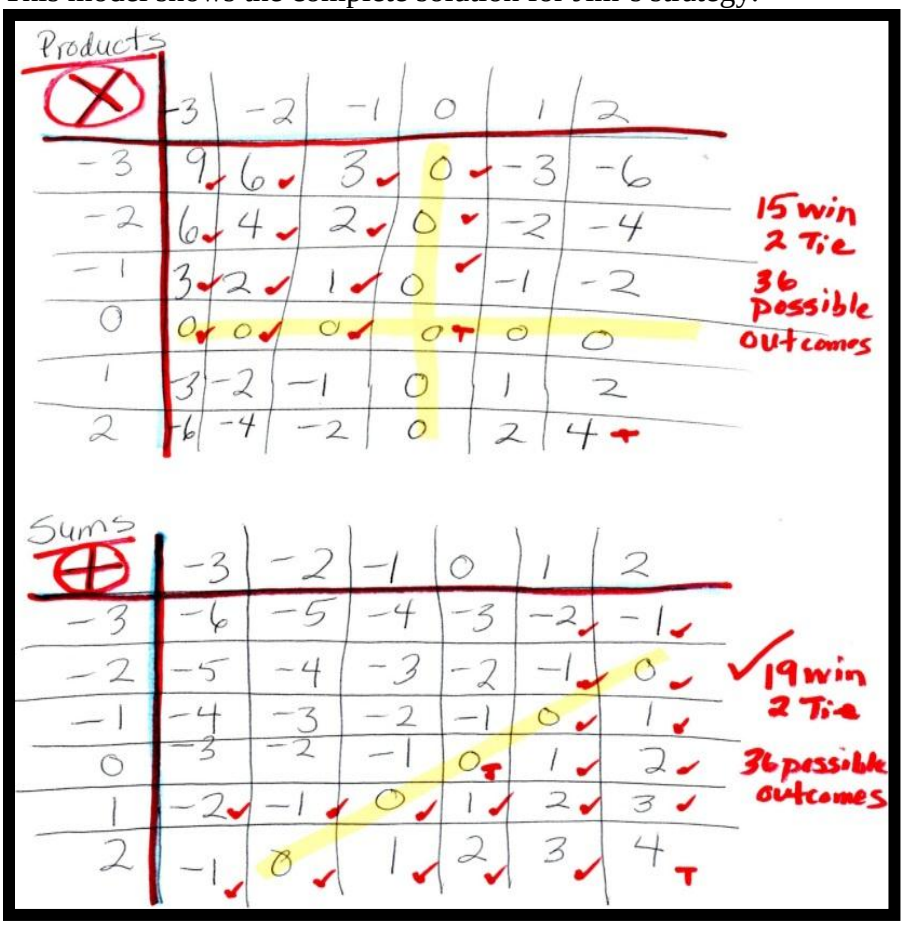
Task adapted from www.mathsolutions.com Integer Dice Game

Solution & Discussion of Student work samples:

Alisha's tree diagram strategy is an appropriate model that could lead to the correct solution, but her work is incomplete. She shows a path to all the outcomes, but does not complete the calculations and identify the expected probabilities for each player.

The model below is similar to Jim's model, however Jim's model was incomplete because it did not indicate which outcome would win each round.

This model shows the complete solution for Jim's strategy.



The model below is similar to Chip's model, however Chip's model has inaccurate integer calculations on all the negative times negative outcomes, so his expected probabilities are also inaccurate.

This model shows the correct solution for Chip's strategy.

1st spinner	2nd Spinner	Sum	Product	Winner
-3	-3	-6	9	×
-3	-2	-5	6	×
-3	-1	-4	3	×
-3	0	-3	0	×
-3	1	-2	-3	+
-3	2	-1	-6	+
-2	-3	-5	6	×
-2	-2	-4	4	×
-2	-1	-3	2	×
-2	0	-2	0	×
-2	1	-1	-2	+
-2	2	0	-4	+
-1	-3	-4	3	×
-1	-2	-3	2	×
-1	-1	-2	1	×
-1	0	-1	0	×
-1	1	0	-1	+
-1	2	1	-2	+
0	-3	-3	0	×
0	-2	-2	0	×
0	-1	-1	0	×
0	0	0	0	Tie
0	1	1	0	+
0	2	2	0	+
1	-3	-2	-3	+
1	-2	-1	-2	+
1	-1	0	-1	+
1	0	1	0	+
1	1	2	1	+
1	2	3	2	+
2	-3	-1	-6	+
2	-2	0	-4	+
2	-1	1	-2	+
2	0	2	0	+
2	1	3	2	+
2	2	4	4	Tie

× $\frac{15}{36}$

+ $\frac{19}{36}$

Tie $\frac{2}{36}$

As student groups play the game they will need to calculate the probability of each player based on the data from their game.

In their post reflection students should give reasoning for their decisions. After the whole group compiles all the experimental data student should see more trials give results closer to the expected results than only using a few trials.

Integer Spinner Game

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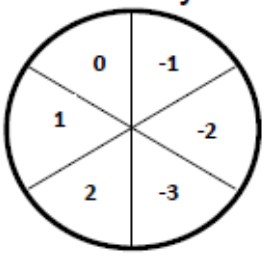
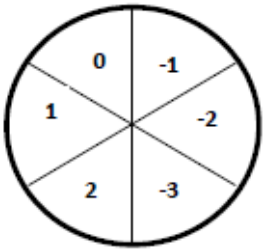


Spinner One



Spinner Two

Use paperclips to spin.

Game Rules:

- Both spinners are spun at the same time for each round.
- **Player A finds the sum** of the two numbers, **Player B finds the product** of the two numbers.
- The winner of the round is the player with the greater result for that round. For example, if Spinner One lands on **-1** and Spinner Two lands on **-2**, then Player A's sum is **-3** and Player B's product is **2**, so player B wins the round.

Your Task:

Represent the sample space with an organized method to determine all the possible outcomes. If you played 100 rounds, use the theoretical probability to predict if Player A or Player B would win more often?

Task adapted from www.mathsolutions.com Integer Dice Game

Integer Spinner Game

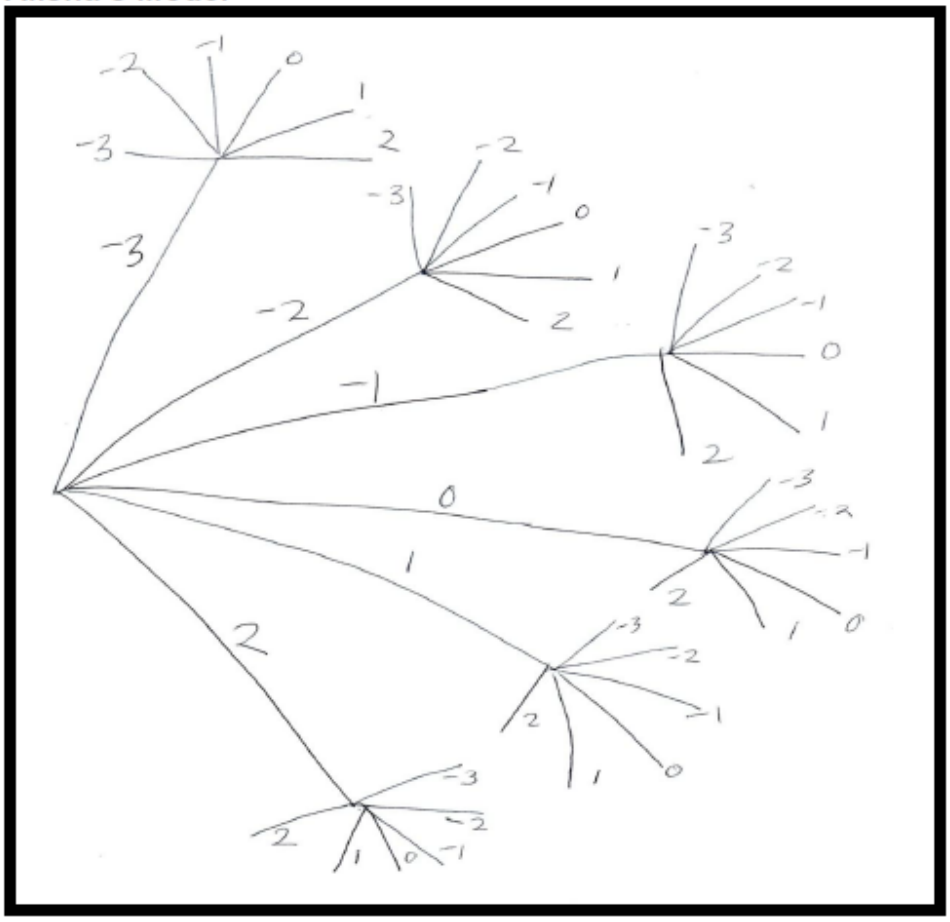
Group Tasks:

1. Analyze the probability models in the student work samples and make suggestions about how each can be improved. As a group come to an agreement about the expected probability for Player A and Player B.

2. With your group, conduct an experiment by playing the **Integer Spinner Game** for as many rounds as you decide are necessary. Compare the frequencies you generate to your probability model and determine if your experimental probability matches what you expected to happen. Is this a fair game?

Task adapted from www.mathsolutions.com Integer Dice Game

Alisha's Model



What Math did Alisha do well?

_____

_____

_____

Is Alisha's model complete?

_____

_____

_____

In what ways might the work be improved?

_____

_____

_____

Task adapted from www.mathsolutions.com Integer Dice Game

Jim's Model

Spinner One

Spinner Two	-3	-2	-1	0	1	2
-3	9	6	3	0	-3	-6
-2	6	4	2	0	-2	-4
-1	3	2	1	0	-1	-2
0	0	0	0	0	0	0
1	-3	-2	-1	0	1	2
2	-6	-4	-2	0	2	4

Spinner one

Spinner Two	-3	-2	-1	0	1	2
-3	-6	-5	-4	-3	-2	-1
-2	-5	-4	-3	-2	-1	0
-1	-4	-3	-2	-1	0	1
0	-3	-2	-1	0	1	2
1	-2	-1	0	1	2	3
2	-1	0	1	2	3	4

What Math did Jim do well?

_____

_____

_____

Is Jim's model complete?

_____

_____

_____

In what ways might the work be improved?

_____

_____

_____

Task adapted from www.mathsolutions.com Integer Dice Game

Chip's Model

1st spinner	2nd Spinner	Sum	Product
-3	-3	-6 ✓	-9
-3	-2	-5 ✓	-6
-3	-1	-4	-3 ✓
-3	0	-3	0 ✓
-3	1	-2 ✓	-3
-3	2	-1 ✓	-6
-2	-3	-5 ✓	-6
-2	-2	-4	-4 T
-2	-1	-3	-2 ✓
-2	0	-2	0 ✓
-2	1	-1 ✓	-2
-2	2	0 ✓	-4
-1	-3	-4	-3 ✓
-1	-2	-3	-2 ✓
-1	-1	-2	-1 ✓
-1	0	-1	0 ✓
-1	1	0 ✓	-1
-1	2	1 ✓	-2
0	-3	-3	0 ✓
0	-2	-2	0 ✓
0	-1	-1	0 ✓
0	0	0	0 T
0	1	1 ✓	0
0	2	2 ✓	0
1	-3	-2 ✓	-3
1	-2	-1 ✓	-2
1	-1	0 ✓	-1
1	0	1 ✓	0
1	1	2 ✓	1
1	2	3 ✓	2
2	-3	-1 ✓	-6
2	-2	0 ✓	-4
2	-1	1 ✓	-2
2	0	2 ✓	0
2	1	3 ✓	2
2	2	4	4 T

What Math did Chip do well?

_____

_____

_____

Is Chip's model complete?

_____

_____

_____

In what ways might the work be improved?

_____

_____

_____

Task adapted from www.mathsolutions.com Integer Dice Game

Post-Task Reflection: <u>Integer Spinner Game</u>

How Did You Work?

Tick the boxes and complete the sentences that apply to your work.

1.) Check one, then complete the sentence below:

☐ My method for finding all the possible outcomes is similar to one of the sample responses. - My method is similar to:

_____
(add name of sample response)

OR

☐ My method is different from **all** of the sample responses.

I prefer (*circle one*) **my method** / **the sample response** method because:

_____

_____

2.) When my group played the spinner game, we played _____ rounds of the game. *(insert number)*
We assumed this was enough rounds to determine the experimental probability because: _____

_____

_____

3.) Check one box then complete the sentence below - After finding the expected probabilities and playing the game, we found:

☐ Our experimental probability from playing the game was ***about the same*** as our expected probability.

☐ Our experimental probability from playing the game was ***very different*** from our expected probability.

I think that when the experimental probability and the expected probability are (circle one) ***about the same*** / ***very different*** this means:

_____

_____

_____

Task adapted from www.mathsolutions.com Integer Dice Game