Beads Under the Cloud

Intermediate/Middle School Grades Problem Solving

Mathematics Formative Assessment Lesson

Designed and revised by Kentucky Department of Education Mathematics Specialists
Field-tested by Kentucky Mathematics Leadership Network Teachers

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Beads Under the Cloud

Intermediate/Middle School Grades

This problem solving Formative Assessment Lesson is designed to be part of an instructional unit. The results of this task should be used to inform the instruction that will take place for the remainder of the unit.

Mathematical goals

This problem solving lesson is intended to help you assess how well students are able to identify patterns (both linear and exponential) in a realistic context: the number of beads of different colors that are hidden behind the cloud. In particular, this problem solving lesson aims to identify and help students who have difficulties with:

- Choosing an appropriate, systematic way to collect and organize data.
- Examining the data and looking for patterns
- Describing and explaining findings clearly and effectively.

Common Core State Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.
8. Look for and make use of repeated reasoning.

This lesson asks students to select and apply mathematical content from across the grades, including the *content standards:*

4-OA: **Generate and analyze patterns.**
5-OA: **Analyze patterns and relationships**
6-EE: **Represent and analyze quantitative relationships between dependent and independent variables.**
8-F: **Use functions to model relationships between quantities.**
F-LE: **Linear, Quadratic, and Exponential Models**

Introduction

This lesson is structured in the following way:

• Before the lesson, students attempt the task individually. You then review their work and formulate questions for students to answer in order for them to improve their work.
• At the start of the lesson, students work individually to answer your questions.
• Next, they work collaboratively, in small groups, to produce a better collective solution than those they produced individually. Throughout their work, they justify and explain their decisions to peers.
• In the same small groups, students critique examples of other students' work.
• In a whole-class discussion, students explain and compare the alternative approaches they have seen and used.
• Finally, students work alone to reflect on their individual & group work.

Materials required

- Each individual student will need one copy of the *Beads under the Cloud* sheet and one copy of the *How did you work?* Sheet.
- Each small group of students will need d a copy of *Sample Responses to Discuss* and whichever samples of student work chosen.

Time needed

Approximately fifteen minutes before the lesson, a one-hour lesson, and ten minutes in a follow-up lesson (or for homework). All timings are approximate. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task:

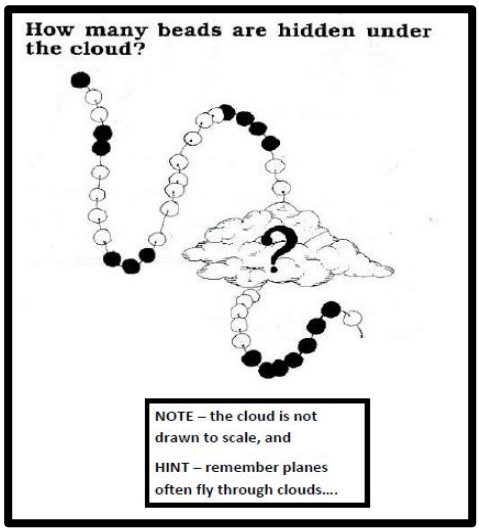
Have the students do this task in class or for homework a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Beads under the Cloud*. Introduce the task briefly and help the class to understand the problem and its context.

Spend fifteen minutes on your own, answering these questions.
Show your work.

Don't worry if you can't figure it out.
There will be a lesson on this material [tomorrow] that will help you improve your work.

Your goal is to be able to answer this question with confidence by the end of that lesson.



It is important that students answer the question without assistance, as far as possible. Students who sit together often produce similar answers, and then, when they come to compare their work, they have little to discuss

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches. The purpose of this is to forewarn you of the issues that will arise during the lesson, so that you may prepare carefully.

We suggest that you do not score students' work. The research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit. We suggest that you write your own lists of questions, based on your own students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.

Common issues - Suggested questions and prompts:

Common Issues	Suggested questions and prompts
Student forgets to look at one of the sets of beads (only looking at white or black, and not both).	• ***How could you simplify this into an easier task?*** • ***What sort of diagram might be helpful?***
Student work is unsystematic. **Student sees the patterns as two separate entities and does not see the relationship between the white and black patterns or how they alternate on the string.**	• ***How do the black beads grow?*** • ***What patterns do you notice?*** • ***What is the same and what is different about the patterns of the black & white beads?***
Student assumes the picture of the cloud is to scale and that not very many beads can fit under the cloud.	• ***What assumptions can you make about the size of the cloud?*** • ***Are all math diagrams always drawn to scale?***
Student writes answers without explanation.	• ***How could you explain/show how you reached your conclusions so that someone in another class understands?*** • ***How can you use words and/or variables to describe the patterns?***
Student does not generalize.	• ***Can you describe a visual pattern in the black beads and the white beads? How could I find out the number of white beads that follow the set of ten black beads? Or the set of 20 black beads?***
Student correctly identifies the pattern for both the black and the white beads.	• ***Think of another way of solving the problem. Is this method better or worse than your original one? Explain your answer. Can you extend your solution to include exponents?***

Suggested lesson outline

Improve individual solutions to the assessment task (10 minutes)

Return your students' work on the *Beads under a Cloud* problem. Ask students to re-read both the *Beads under the Cloud* problem and their solutions. If you have not added questions to students' work, write a short list of your most common questions on the board. Students can then select a few questions appropriate to their own work and begin answering them.

> ***Recall what we were working on previously. What was the task?***

Draw students' attention to the questions you have written.

> ***I have read your solutions and I have some questions about your work.***
>
> ***I would like you to work on your own to answer my questions for ten minutes.***

Collaborative activity:

Organize the students into small groups of two or three. In trials, teachers found keeping groups small helped more students play an active role.

Students should now work together to produce a joint solution.

> ***Put your solutions aside until later in the lesson. I want you to work in groups now.***
>
> ***Your task is to work together to produce a solution that is better than your individual solutions.***

You as the teacher have two tasks during small-group work, to note different student approaches to the task, and to support student problem solving.

Note different student approaches to the task

Notice how students work on finding the patterns for the white and black beads. Notice what strategies they use and how they organize their data. Do they use tables or pictures or some other way to organize their work? Older students might introduce algebra. If they do use algebra, note the different formulations of the functions they produce, including incorrect versions, for use in whole-class discussion. You can use this information to focus the whole-class plenary discussion towards the end of the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. If several students in the class are struggling with the same issue, you could write a relevant question on the board. You might also ask a student who has performed well on one part of the task to help a student struggling with that part of the task.

The following questions and prompts would be helpful:

> ***What information have you been given?***
>
> ***What do you need to find out?***
>
> ***What changes in the diagram? What stays the same?***
>
> ***How will you write down your pattern?***
>
> ***Why do you think your conjecture might be true?***

Teachers who have used this activity noted that the students sometimes lose sight of both color patterns and how the patterns are related. If this issue arises in your class, help the student to focus his or her attention on both the white and black bead patterns and the connections between the patterns.

How are the white beads changing?
How are the black beads changing?
What effect does the change of the white bead have on the change in the black beads?

You may find that some students do not work systematically when organizing their data.

What can you do to organize your data to show both patterns?

If students have found function rules or equations, focus their attention on improving explanations, or exploring alternative methods.

How can you be sure your explanation works in all cases?
Ask another group if your argument makes sense.
Show me that this equation works.

Some stronger explanations are shown in the *Sample Responses to Discuss*.

Make a note of student approaches to the task

Give each small group of students a copy of the *Sample Responses to Discuss*. Choose the samples of student work that match your students' level of understanding. Solutions by Rick & Curtis should only be given to more advanced student groups. Rick shows algebraic equations for both linear & exponential patterns and Curtis uses an incorrect pattern that might be confusing to some students. Display the following questions on the board or project the provided sheet: *Analyzing Student responses to discuss*.

Describe the problem solving approach the student used.
You might, for example:
• Describe the way the student has organized the data.
• Describe what the student did to calculate number of beads under the cloud.
Explain what the student could do to make his or her solution correct or clearer if they calculated correctly.

This analysis task will give students an opportunity to evaluate a variety of alternative approaches to the task, without providing a complete solution strategy.

During small-group work, support student thinking as before. Also, check to see which of the explanations students find more difficult to understand. Identify one or two of these approaches to discuss in the plenary discussion. Note similarities and differences between the sample approaches and those the students took in small-group work.

Plenary whole-class discussion comparing different approaches (20 minutes)

Organize a whole-class discussion to consider different approaches to the task. Discuss some of the different approaches used in the sample work and ask students to comment on their strengths and weaknesses. You may also want to compare students' own work with the sample student responses. Did any group use a similar method to one of the samples? What was the same about the work?

What was different about the work? What were the errors in the sample student responses and in what ways did analyzing the responses enable students to identify errors in their own work?

You may want to use the student work samples and/or questions in the *Common Issues* table to support this discussion.

The intention is for you to focus on getting students to understand the methods, rather than either numerical or algebraic solutions. Focus your discussion on parts of the two small-group tasks students found difficult.

Let's stop and talk about different approaches.

Ask the students to compare the different solution methods.

Read through your original responses and think about what you have learned this lesson.

Which approach did you like best? Why?

Which approach did you find it most difficult to understand?

Sami, your group used that method. Can you explain that for us?

Individual reflection (10 minutes) – possible homework

Once students have had a chance to discuss the sample responses as a whole class, distribute the questionnaire *How Did You Work?* Ask students to spend a couple of minutes, individually, answering the questions.

Think carefully about your work on this lesson and the different methods you have seen and used.

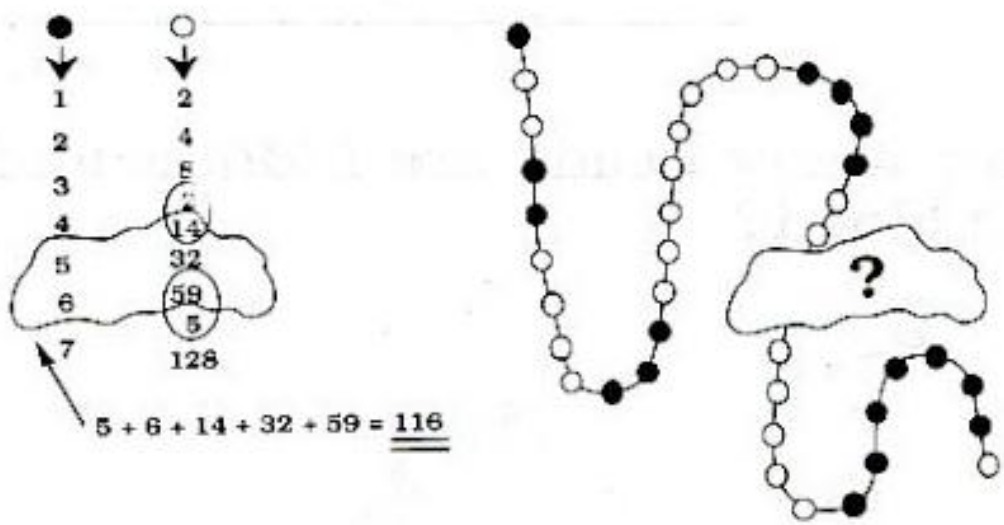
If you are running out of time, you could schedule this activity for the next lesson or for homework.

This Formative Assessment Lesson was created around a task taken from *Mathematics for Elementary Teachers (A Contemporary Approach) – Musser-Burger.*

Solution

Problem: How many beads are hidden under the cloud?

Solution: What patterns do the black beads and white beads make? What's hidden under the cloud: black beads, white beads, or some of both?



Analysis of *Student Responses to Discuss*

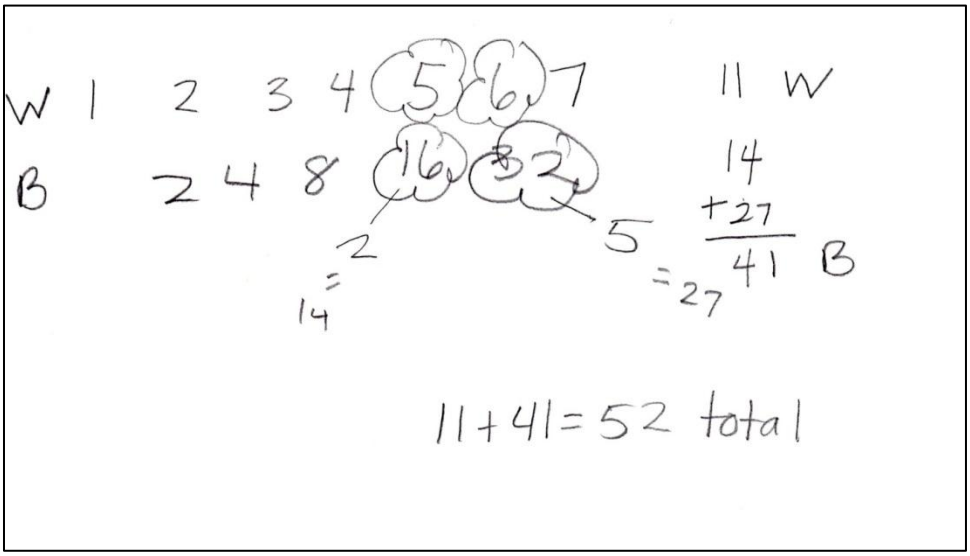
Timothy's Method

Timothy made a list of the number of black and white beads. He found the correct pattern of both sets of beads. However, because he did not systematically organize the data, Timothy omits the 64 white beads.

Timothy's strategy of only counting the beads under the cloud is correct and he knew to subtract the beads from the pattern that were showing.

Timothy could use a table to better organize his data. By doing this, he could see that the 16 black beads correspond to the 4 white beads, the 32 black beads correspond with the 5 white beads, etc.

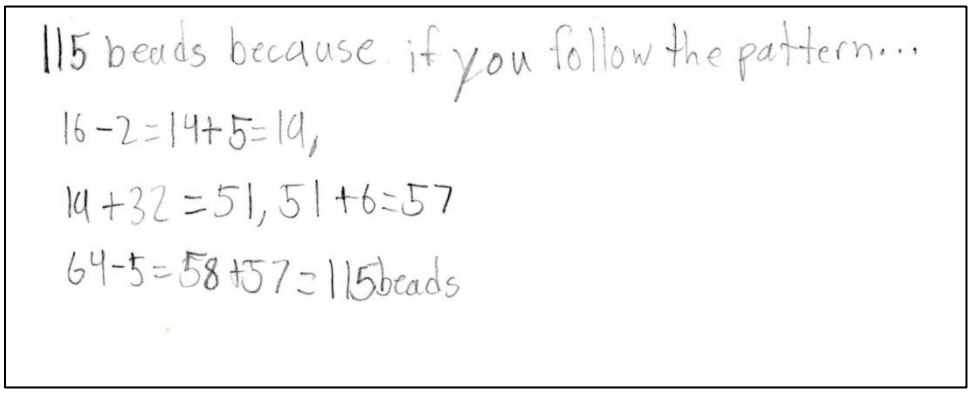
Timothy's Solution



Hannah's Method

Hannah followed the pattern correctly because she uses the numbers 16, 5, 32, 6, and64 in her calculations even though she did not show the data in an organized list. She miscalculated 64-5 and used 58 instead of 59 in her final sum of beads under the cloud.

Hannah's Solution

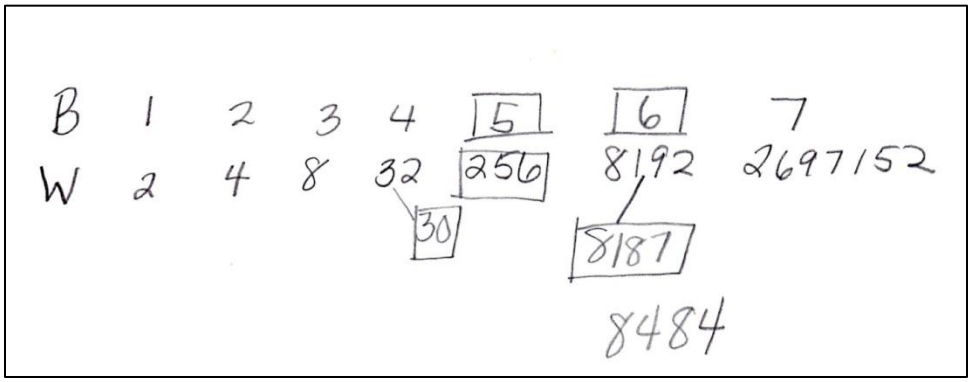


Curtis' Method

Curtis correctly identified the pattern of the black beads. He knew to subtract the 2 beads from 32 and 5 beads from 8,192. This sample should only be given to more advanced student groups.

Curtis did not generalize the pattern of the white beads correctly. He multiplied the two previous numbers to generate the next number of beads. If he were asked to explain how this generalization could be applied to grow from 2 white beads to 4 white beads, he might realize that this rule does not apply to all numbers in the pattern.

Curtis' Solution

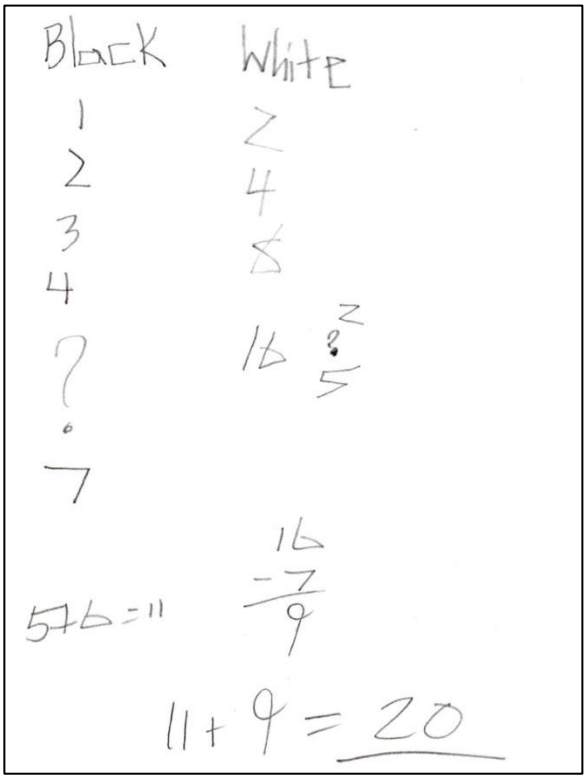


Tony's Method

Tony generated both patterns correctly and attempted to organize the data in a systematic way. However, he does not see the correspondence between the black and white beads. Even though he knew to add the 5 and 6 black beads that he represented by the question mark in his table, he doesn't add the 32 and 64 white beads that correspond to these numbers.

Tony could continue his pattern of the white beads to possibly see that there would be more white beads to add in to the total of beads under the cloud.

Tony's Solution



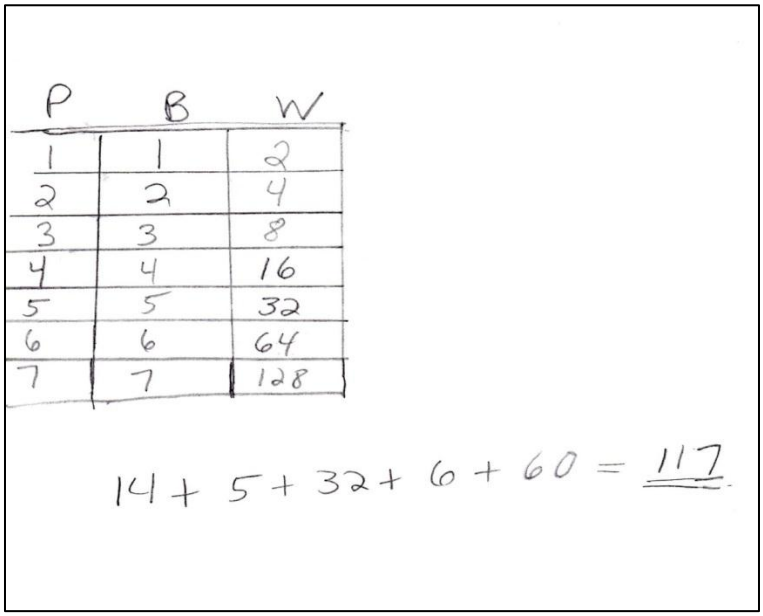
Claire's Method

Claire correctly generated both patterns. She has labeled the sequence of beads with a term number.

Claire uses the correct strategy for determining how many beads are under the cloud even though she does not show how she arrived at the 14 and 60.

Claire's total number of beads under the cloud was incorrect because she subtracted 4 from 64 instead of 5.

Claire's Solution



P	B	W
1	1	2
2	2	4
3	3	8
4	4	16
5	5	32
6	6	64
7	7	128

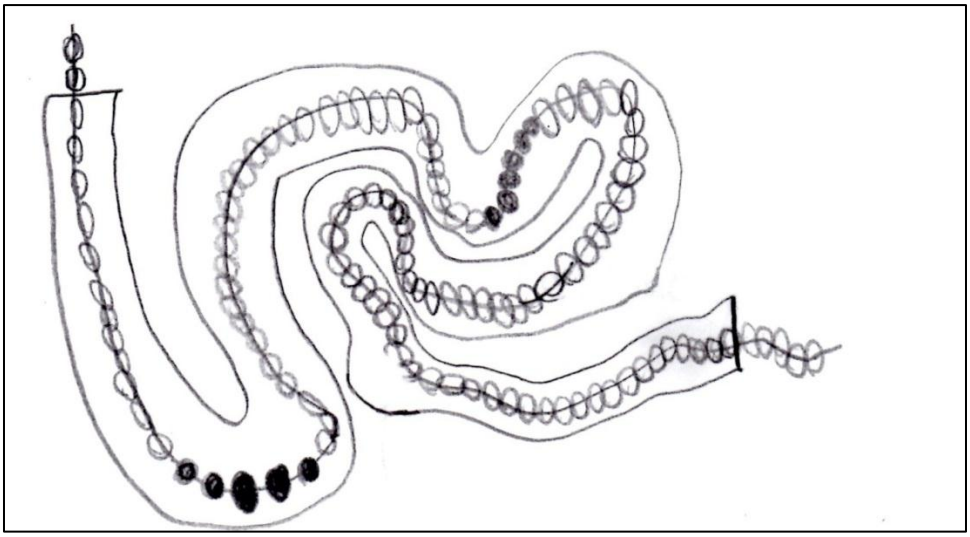
$14 + 5 + 32 + 6 + 60 = \underline{\underline{117}}$

Victoria's Method

Victoria drew a picture to demonstrate the pattern of the black and white beads hidden by the cloud.

Victoria does not have a numerical answer about how many beads are hidden but does have 5 black beads, 32 white beads, 6 black beads, and 64 white beads drawn. She has 59 of the 64 white beads under the cloud. Victoria has not drawn the correct number of white beads under the cloud (14) but does have 14 white beads drawn.

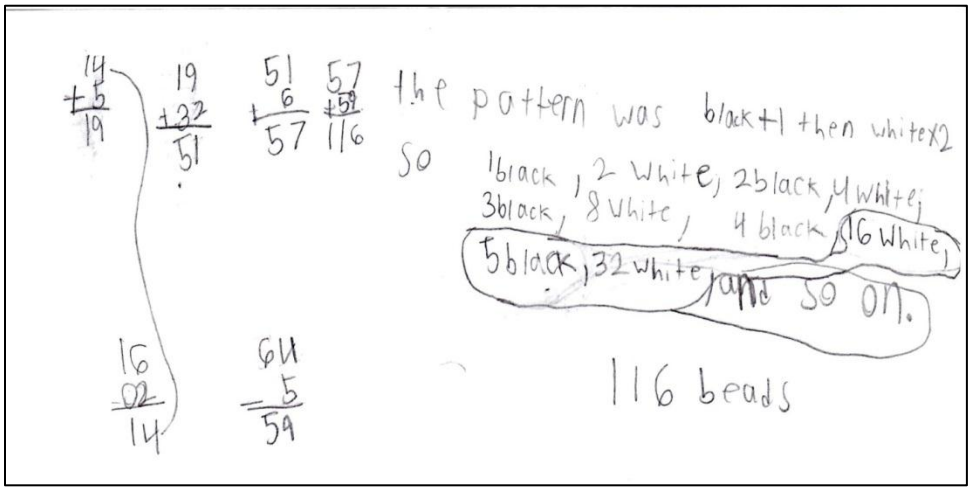
Victoria's Solution



Sam's Method

Sam correctly calculated the number of beads hidden under the cloud. He listed the pattern and showed his calculations. He attempts generalizing a rule to repeat the pattern but does not identify the relationship between the corresponding terms.

Sam's Solution

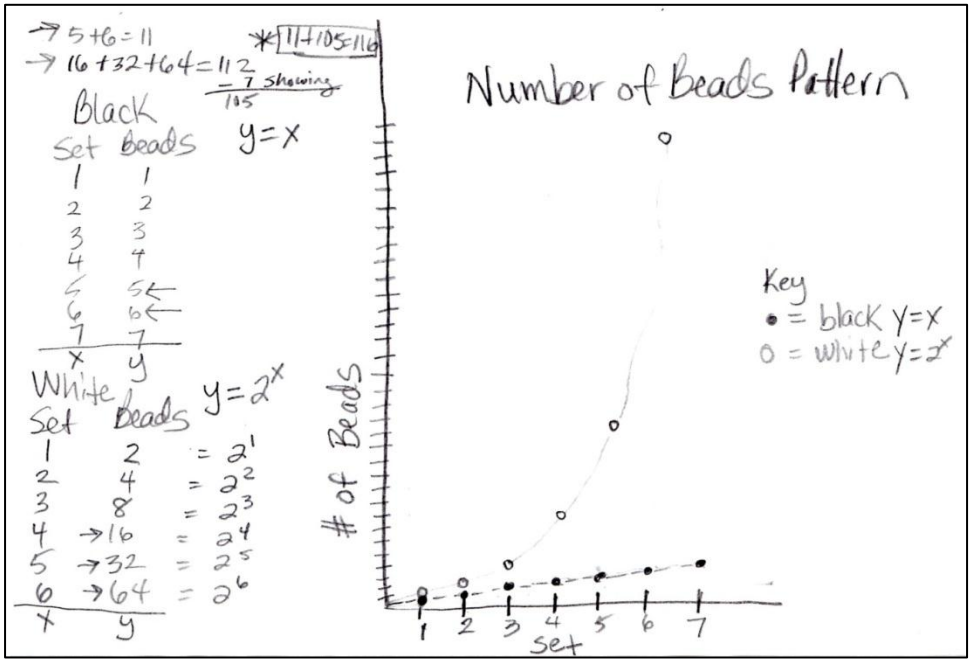


Rick's Method

Rick uses algebra to determine the patterns of the beads. He generated the rule to determine the next numbers in the pattern. Rick recognized the linear relationship of the black beads and the exponential relationship of the white beads. He also graphed the data. This student work sample should only be given to more advanced student groups.

Rick subtracted the correct number of white beads showing in the 4th and 6th set of beads.

Rick's Solution



How many beads are hidden under the cloud?

NOTE – the cloud is not drawn to scale, and

HINT – remember planes often fly through clouds….

Sample Responses to Discuss

Here is some work on *Beads under the Cloud* from students in another class.

For each piece of work:

1. Write the name of the student whose solution you are analyzing.
2. Describe the problem solving approach the student used.

   For example, you might:

   - Describe the way the student has organized the data.
   - Describe what the student did to calculate a number of beads under the cloud.

3. Explain what the student needs to do to complete or correct his or her solution.

_____’s_ Solution

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_____’s_ Solution

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_____’s_ Solution

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How Did You Work?

Post-Task Reflection: ***Beads Under the Cloud***

Tick the boxes and complete the sentences that apply to your work.

1.) Check one, then complete the sentence below:

☐ Our group work was better than my own individual work.

OR

☐ My own individual work is better than our group work.

I prefer (*circle one)* **our method** / **my method** because:

_____

_____

2.) Check one, then complete the sentence below:

☐ Our method is similar to: _____
(add name of sample response)

OR

☐ Our method is different from **all** of the sample responses.

I prefer (*circle one)* **our method** / **the sample response** method because:

_____

_____

3.) Check one and complete the sentence:

☐ We checked our method by: _____

OR

☐ We could check our method by: _____

Collaborative work

(1) Share your method with your partner(s) and your ideas for improving your individual solution.

(2) Together in your group, agree on the best method for completing the problem.

(3) Produce a poster, showing a joint solution to the problem.

(4) Make sure that everyone in the group can explain the reasons for your chosen method, and describe any assumptions you have made.

(5) Check your work.

Timothy's Solution

W 1 2 3 4 5 6 7 11 W

B 2 4 8 16 32 14

+27

41 B

2 = 14

5 = 27

11+41=52 total

Hannah's Solution

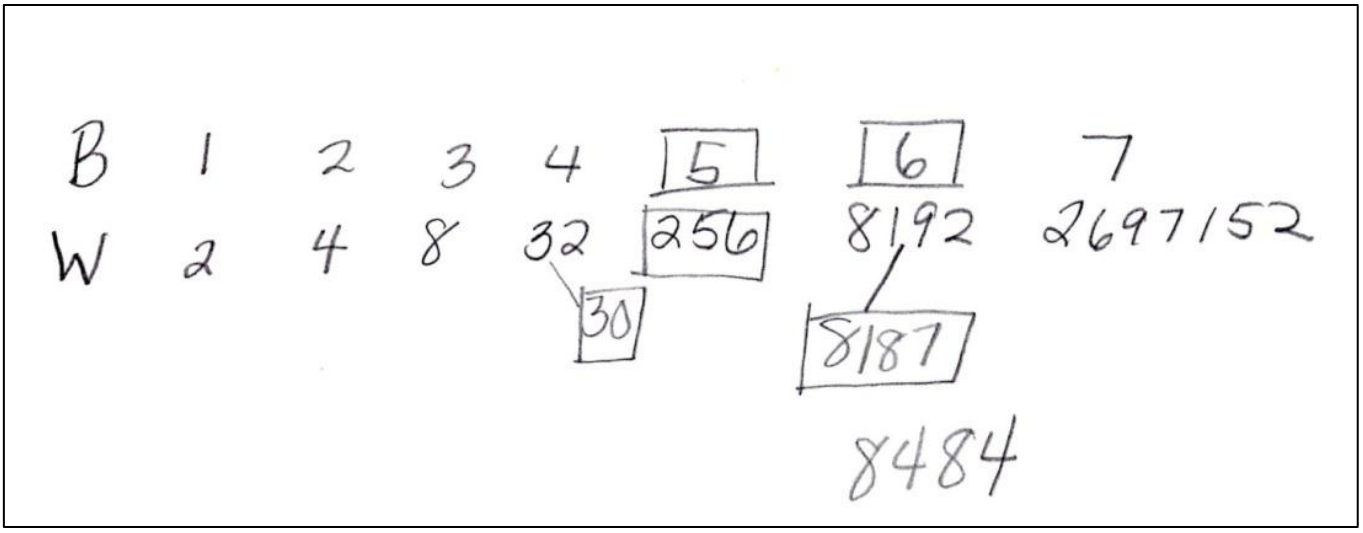
115 beads because if you follow the pattern...

16-2=14+5=19,

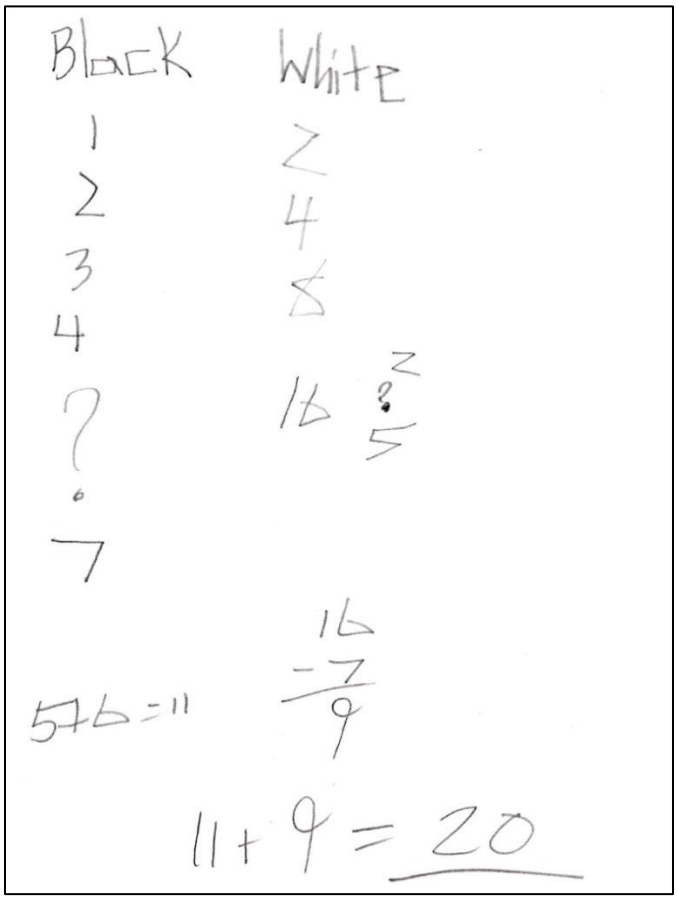
19+32=51, 51+6=57

64-5=58+57=115beads

Curtis' Solution



Tony's Solution



Claire's Solution

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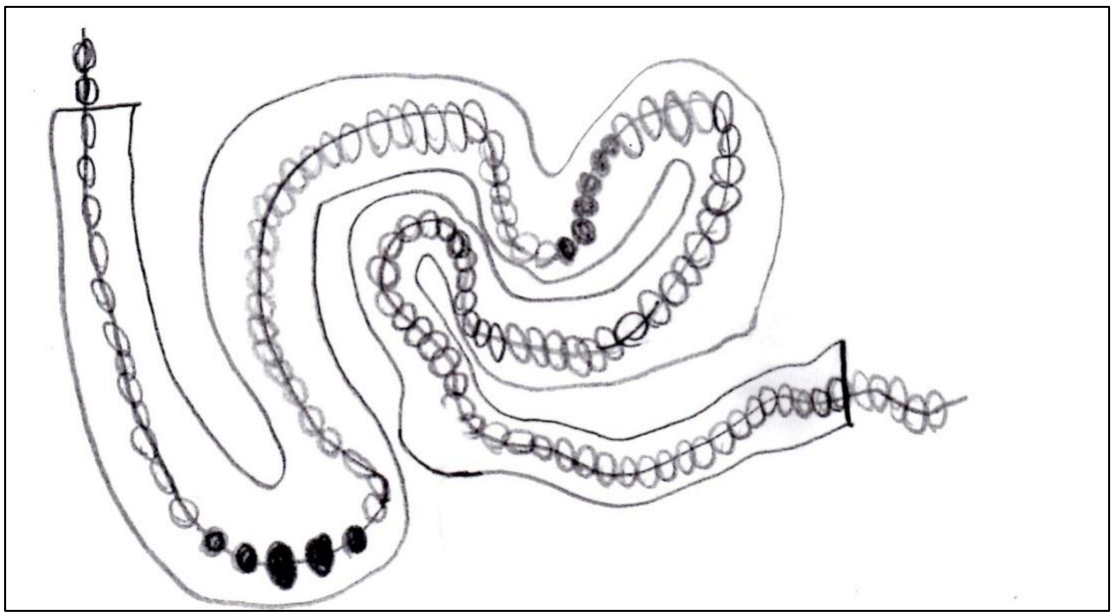
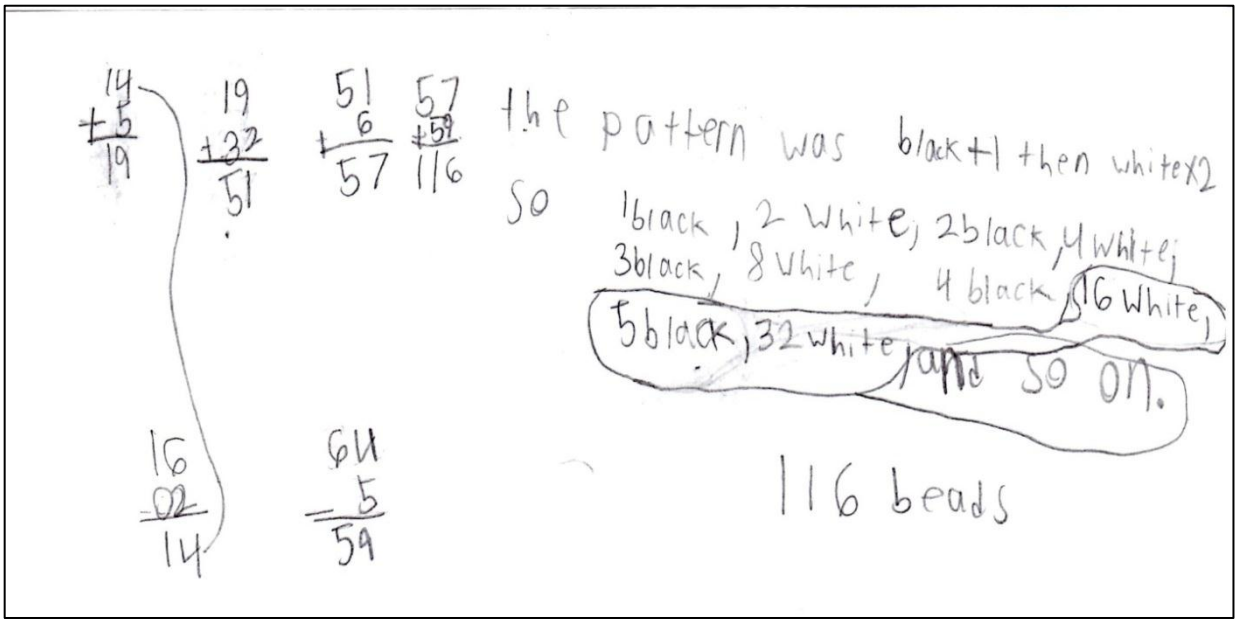
Victoria's Solution



Sam's Solution



Rick's Solution